Constructing a volatility index for Brazil's FX Rate

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Outline

- Motivation
- Investor's fear gauge
- Assumptions
- Variance Swap
- FXvol

Food for your thoughts

The implied volatility is just
'the wrong number to put in the wrong (Black) formula to get the right price'

Ricardo Rebonato

Motivation

- The construction of implied volatility indices has been primarily motivated by the increasing need to create derivatives on volatility
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Investor's fear gauge

- Authors found a negative relationship between returns and Volatility Indexes;
- A possible explanation for this is that the demand for puts (calls) increases when the market goes down (up);
- Hence, they interpret the Volatility Indexes as *the investor's fear gauge*;
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Assumptions

- H1 - The volatility index is mathematically equivalent to a Variance Swap (Demeterfi *et ali* (1999);
- H2 - The underlying asset is a diffusion;
- H3 - Replicating Portfolio (Market is complete)
- H4 - In a risk-neutral World, the expected asset return is the "risk free" rate

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The easiest way to trade variance is to use *variance swaps*
Traditionally it is traded directly between two parties in over the counter (OTC) markets and its payoff at expiration is given by

$$N \times (\langle X \rangle_T - \sigma^2(T))$$

where $\langle X \rangle_T$ is the quadratic variation of the asset price process quoted on an annual basis and N is the notational amount of swap.

Under H1 the fair value of variance swap with maturity T is :

$$\sigma^2(T) = \mathbb{E}^{\mathbb{Q}} [\langle X \rangle_T]$$

Assuming H2, the underlying asset follows a diffusion process (no jumps):

$$\frac{dS_t}{S_t} = \mu(t, \cdots)dt + \sigma(t, \cdots)dw_t \tag{1}$$

By applying Ito's lemma to $\log S_t$, we have:

$$d\left(logS_t\right) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dw_t \tag{2}$$

Subtracting term (2) from (1):

$$\frac{dS_t}{S_t} - d\left(logS_t\right) = \frac{\sigma^2}{2} dt \tag{3}$$

Summing equation (3) over all times from 0 to T and applying the Risk Neutral expectation, we obtain the continuously-sampled variance

$$\sigma^2(T) = \frac{2}{T}\mathbb{E}^{\mathbb{Q}}\left[\int_0^T \frac{dS_t}{S_t} - log\frac{S_T}{S_0}\right]$$

Assumption H4 implies:

$$\mathbb{E}^{Q}\left[\int_0^T \frac{dS_t}{S_t}\right] = rT$$

Here r is the risk-free discount rate corresponding to the expiration date T;

The second term represents a static short position in a contract which, at expiration, pays the logarithm of the total return.

Assumption H3 implies we can replicate $-log\frac{S_T}{S_0}$ using:

$$log\left(\frac{S_T}{S_0}\right) = log\left(\frac{S_T}{S_*}\right) + log\left(\frac{S_*}{S_0}\right)$$

$$-log\left(\frac{S_T}{S_*}\right) = -\frac{S_T - S_*}{S_*} + \int_0^{S_*} \frac{1}{K^2} max\left(K - S_T, 0\right) dK + \int_{S_*}^{\infty} \frac{1}{K^2} max\left(S_T - K, 0\right) dK \quad (4)$$

equation (4) represents the decomposition of a log payoff (Carr and Wu (2006)) into a portfolio consisting of:

- a short position in $\left(\frac{1}{S_*}\right)$ forward contracts struck at S_*;
- a long position in $\left(\frac{1}{K^2}\right)$ put options struck at K, for all strikes at interval $[0, S_*]$;and
- a similar long position in $\left(\frac{1}{K^2}\right)$ call options struck at K, for all strikes at interval $[S_*, \infty]$.

In practice, only a small set of discrete option strikes is available, so we replace the integral for a sum

$$\sigma^2(T) = \frac{2}{T}[-rT - \left(\frac{S_0}{S_*}e^{rT} - 1\right) - log\left(\frac{S_*}{S_0}\right) \quad (5)$$

$$+ \quad e^{rT}\left(\sum_{0}^{S_*}\frac{\Delta K}{K^2}P(K)dK\right) \quad (6)$$

$$+ \quad \left(\sum_{S_*}^{S_N}\frac{\Delta K}{K^2}C(K)dK\right)] \quad (7)$$

- Finally, the FXVol is computed using all available FX options traded at BM&FBOVESPA for the first two maturities, T and T_1, with $T_1 > T$
- Linear interpolation + square root ⇒

$$FXvol = \sqrt{\left(T\sigma^2(T)\left[\frac{DU_{T1} - DU_{21}}{DU_{T1} - DU_T}\right] + T_1\sigma^2(T_1)\left[\frac{DU_{21} - DU_T}{DU_{T1} - DU_T}\right]\right) \times \frac{DU_{252}}{DU_{21}}}$$

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- FXVol has been available on a daily basis since 2005

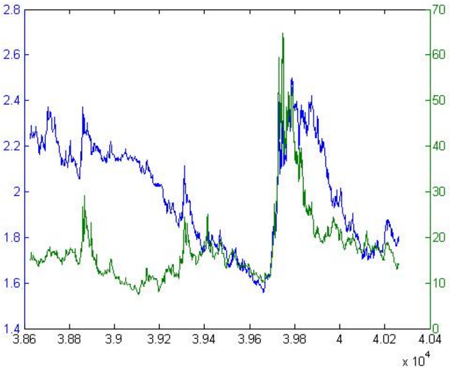

Figure: **BRL/USD (green) and FXvol (Blue)**

	FXvol	USD/BRL	Ibovespa
FXvol	1	0.35	-0.30
BRL/USD		1	-0.43
Ibovespa			1

Table: **Correlation Matrix**

- Portfolio diversification with FXVol is an alternative

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- Two mutual Funds invested 100.000 BRL in 2006;
- Ibovespa and FXvol Benchmarks;

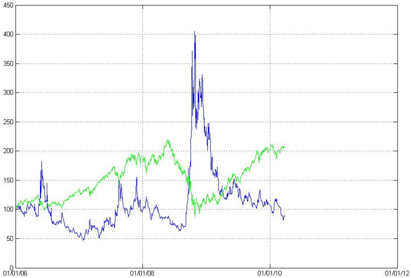

Figure: **Two investing strategies - Ibovespa (green)and FXvol (Blue)**

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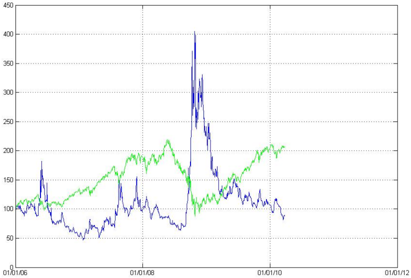

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